Refinement and iteration methods such as multigrids are also considered. Extensions include a short survey of hierarchic (p-methods) elements; error estimates are recalled but practical implementation of such a method is left as an exercise, although it is not yet a very familiar one.

The next chapter, written by A. Sherman, deals with the solution of sparse systems of linear equations. Methods for positive definite systems are given first: sparse methods (symbolic factorization, band and envelope methods) and their related optimization problems (band reduction, nested dissection, minimum-degree ordering). Next, iterative methods are recalled (conjugate gradient with preconditioning). This is followed by a section on "out of core" implementations of symmetric Gaussian elimination (band and frontal methods). Finally, general nonsingular systems are briefly considered.

The next chapter is about eigenvalue problems. The principal iterative methods considered are the subspace iteration method and the Lanczos method. Givens' and Householder's reductions are also given. Methods are described mainly in the case of the classical eigenvalue problem, but a short section recalls how to extend these methods to the generalized eigenvalue problem. Error estimates and applications of eigenvalue/eigenvector computations are given: mainly modal superposition and matrix conditioning.

Chapter 5 introduces time-dependent problems. Parabolic semidiscretization (forward, central and backward difference) is studied by means of eigenexpansions. Second-order (wave) equations are very briefly considered. More details are given for convection diffusion type problems. Splitting methods, including ADI, are recalled and there is a brief analysis of stability and errors. Mention is made of moving mesh methods.

The book includes more than 170 references and an appendix on interpolation and quadrature. If we except the first chapter, the book is actually quite self-contained. It is well written and pleasantly presented, with numerous examples. It will be of help for both researchers and students in need of implementing or understanding actual coding of the FEM. The only serious shortcoming is that the most useful algorithms for second-order (wave) problems are not given; maybe the authors are planning to include that in their volume on solid mechanics.

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13[65N30].—VIDAR THOMÉE, *Galerkin Finite Element Methods for Parabolic Problems*, Lecture Notes in Mathematics 1054, Springer-Verlag, Berlin, 1984, v + 235 pp., 24 cm. Price $11.00.

From the author's preface:

"The purpose of this work is to present, in an essentially self-contained form, a survey of the mathematics of Galerkin finite element methods as applied to parabolic problems. The selection of topics is not meant to be exhaustive, but rather reflects the author's involvement in the field over the past ten years. The goal has been mainly pedagogical, with emphasis on collecting ideas and methods of analysis in simple model situations, rather than on pursuing each approach to its limits. The notes thus summarize recent developments, and the reader is often referred to the literature for more complete results on a given topic. Because the formulation and analysis of Galerkin methods for parabolic problems are generally based on facts concerning the corresponding stationary elliptic problems, the necessary elliptic results are included in the text for completeness."

This book is not intended to be a guide for the practitioner on how to solve parabolic problems by Galerkin methods. Rather, it shows what a rigorous mathematical analysis can contribute to a qualitative understanding of the behavior of the errors as the mesh sizes become small. The material is presented in 14 more or less self-contained chapters. It includes results for the usual semidiscrete and fully discrete approximation, as well as for nonstandard discretizations, such as the discontinuous Galerkin method in time, the H^1- and H^{-1}-methods, and a mixed method. Special attention is focused on the case of nonsmooth initial data. Throughout, the objective is to find asymptotically "optimal" estimates of the discretization error in terms of the mesh sizes, under minimal assumptions on the smoothness of the solution.

The presentation is mathematically clear and easy to read. It clearly benefits from the fact that the author's own research has contributed to all of the topics discussed. His search for the most appropriate proof techniques results, for instance, in three different approaches to error estimates for semidiscrete Galerkin approximations.

The book is a welcome addition to the theoretically oriented literature on modern numerical methods for time-dependent problems, an area which is not yet very rich in monographs and textbooks.

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14[65G10].—H. RATSCHEK & J. ROKNE, *Computer Methods for the Range of Functions*, Ellis Horwood Series, Mathematics and its Applications, Wiley, New York, 1984, vi + 168 pp., 23 cm. Price $42.95.

This book deals with one of the most central problems in the mathematics of computation: finding the range of values of a function by efficient computational methods. This includes, for example, finding the global minimum and maximum over a given domain, and finding bounds on all sorts of expressions for errors in numerical methods.

The book is an up-to-date account of methods based on interval analysis and contains much original research by the authors. The centred form plays a large role, and many beautiful results are obtained by the authors for this useful tool.

There is no other comparable work in existence and this book should last as an indispensable reference for a long time to come. It is very clearly written and contains a wealth of information of use to anyone concerned with the mathematics of computation.